LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIFTH SEMESTER – November 2009

MT 5505 - REAL ANALYSIS

Date & Time: 3/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL questions **($10 \times 2 = 20$)**

1. Give an example of a subset of real numbers which is not order complete.
2. Show that the sets Z and N are similar.
3. Define discrete metric space.
4. What is meant by a perfect set?
5. Show that limit of a sequence is unique.
6. When do you say that a sequence has a removable discontinuity at a point c?
7. Define local minimum and local maximum of a function at a point.
8. State Lagrange's mean value theorem.
9. If f and α are bounded real valued functions on $[a, b]$, when do you say that f is Riemann integrable with respect to α on $[a, b]$.
10. Define limit superior and limit inferior of a sequence $\{a_n\}$.

SECTION – B

Answer any FIVE questions. **($5 \times 8 = 40$)**

11. Show that every subset of a countable set is countable
12. Show that a subset E of a metric space (X, d) is closed in X if and only if it contains all its limit points.
13. Prove that a closed subset of a compact metric space is compact.
14. Show that a sequence $\{x_n\}$ in a metric space (X, d) converges to p if and only if every subsequence of $\{x_n\}$ converges to p.
15. State and prove Roll's theorem.
16. If f, g are functions of bounded variation on $[a, b]$, show that $f + g$, fg are also of bounded variation on $[a, b]$.
17. Suppose $f \in R(\alpha)$ on $[a, b]$. Show that $\alpha \in R(f)$ on $[a, b]$ and
$$\int_a^b f d\alpha + \int_a^b \alpha df = f(b)\alpha(b) - f(a)\alpha(a).$$
18. Let $\{a_n\}$ be a real sequence. Show that $\{a_n\}$ converges to l if and only if
$$\lim\inf a_n = \lim\sup a_n = l.$$

SECTION – C

Answer any TWO questions **($2 \times 20 = 40$)**

19. (a) If $\mathcal{F}$ is a countable family of countable sets, show that $\bigcup_{F \in \mathcal{F}} F$ is also countable.

    (b) Show that finite intersection of open sets is open in a metric space X. What about arbitrary intersection of open sets? Justify your answer.

20. (a) Show that every convergent sequence is a Cauchy sequence but not conversely

    (b) Define uniformly continuous function. Let X be a compact metric space, Y be a metric space and $f: X \to Y$ be continuous. Show that f is uniformly continuous.

21. (a) State and prove Taylor's theorem.

    (b) State and prove Chain rule for differentiation.

22. (a) Suppose $f \in R(\alpha)$ on $[a, b]$, α differentiable on $[a, b]$ and α' continuous on $[a, b]$. Show that $\int f\alpha' dx$ exists and $\int_a^b f d\alpha = \int_a^b f\alpha' dx$.

    (b) Show that R is a complete metric space.

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